Interference of Light

Interference refers to the effects obtained when two or more light waves of same frequency, comparable amplitude and with a constant phase difference combine.

In interference, redistribution of energies results in such a way that certain areas become more intense /brighter while other areas become less intense or darker.

Principle of Superposition

Principle of superposition is that the net displacement of the medium at any point in space or time, is the sum of the displacements of individual waves. The principle of superposition is applicable to two or more waves travelling through the same medium at the same time.

Coherence

Two light sources are coherent if they emit light waves of

- ✓ Same frequency
- ✓ Same wavelength
- ✓ comparable amplitude
- ✓ Constant phase relationship

Why two independent light sources cannot be coherent?

Individual atoms or molecules at exited stat have a lifetime of typically 10ns. Atoms emit light at random manner in this 10ns lifetime. Thus, waves emitted by two independent sources of light can never have same phase or a constant phase difference.

Coherence Length & Coherence Time

Coherence Length : it is the average length of wave packet with phase correlation, emitted by a source,

$$l_o = \frac{{\lambda_o}^2}{\Delta\lambda}$$

where λ_0 is the centre wavelength of the light source and $\Delta\lambda$ is its spectral line width.

Coherence Time

$$\tau_o = \frac{l_o}{c}$$

where c is the velocity of light in vacuum.

Optical Path Length

It is the distance a ray would have travelled in free space with the time it travelled in a medium.

Optical Path Length = Refractive Index x Geometric Distance

$$OPL = \mu d$$

Phase Difference

Phase of a particle represent its state of vibration. Phase difference can be expressed in angle as a fraction of 2π.

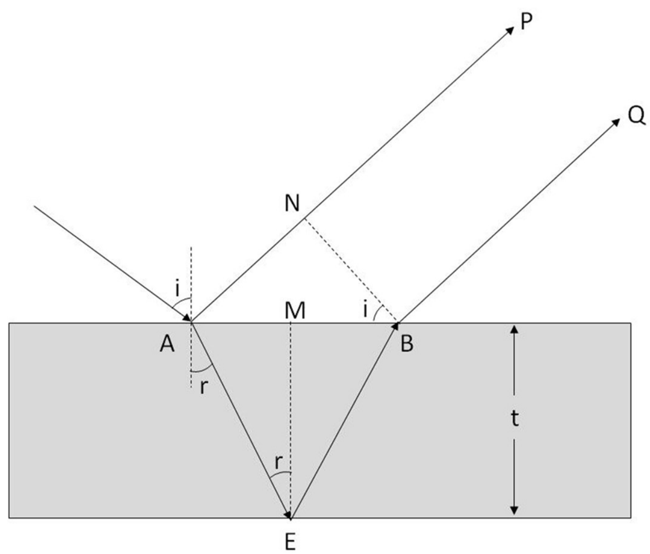
$$\phi = \frac{2\pi}{\lambda} d$$

Interference in Thin Films –Reflected System

Consider a thin transparent film of uniform thickness 't' and refractive index μ. When light of wavelength λ falls on the first surface of the thin film, a portion of the incident wave is partially reflected and other portion is transmitted into the thin film. The transmitted portion is then reflected from a second surface and emerges out of the film.

Two waves emerging from the thin film are

(1) wave reflected from top surface of film (ray AP) and
(2) wave reflected from bottom surface of thin film (ray AEBQ)



These waves have different optical path lengths that is determined by the width of the film. The two waves will eventually interfere and the interference pattern is observed at PQ.

Path difference between the rays AP and AEBQ is

$$\Delta = \mu(AE + EB) - AN$$

$$AE = EB, \quad AE + EB = 2AE$$

$$\Delta = 2\mu\, AE - AN \qquad (1)$$

In triangle AEM,

$$\cos r = \frac{EM}{AE}$$

$$\cos r = \frac{t}{AE}$$

$$AE = \frac{t}{\cos r} \qquad (2)$$

$$\tan r = \frac{AM}{EM}$$

$\tan r = \frac{AM}{t}$

$AM = t \tan r$ (3)

In triangle ANB,

$\sin i = \frac{AN}{AB} = \frac{AN}{2\,AM}$

$AN = 2\,AM \sin i$

Using equation (3)

$AN = 2\,t \tan r \sin i$

Fromm Snell's law

$\mu = \frac{\sin i}{\sin r}$

$\sin i = \mu \sin r$

$AN = 2t \tan r\, \mu \sin r$

$AN = 2\mu t\, \frac{\sin}{\cos r}$ (4)

Using equations (2) and (4) in eqn(1)

$\Delta = 2\mu \frac{t}{\cos r} - 2\mu t\, \frac{\sin 2r}{\cos r}$

$\Delta = 2\mu \frac{t}{\cos r}(1 - \sin 2r)$

$\Delta = 2\mu \frac{t}{\cos r}\, \cos 2r$

$\Delta = 2\mu t \cos r$

This is known as Cosine Law of Interference

Considering the phase change at boundary of optically rarer to denser medium, the path difference between rays getting reflected from upper and lower surfaces of a thin film is

$\Delta = 2\mu t \cos r - \frac{\lambda}{2}$

Condition for Constructive Interference

For constructive interference the path difference between the interfering waves should be an integral multiple of λ

$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$

$2\mu t \cos r = n\lambda + \frac{\lambda}{2}$

$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$ where n=0,1,2,3,…..

Condition for Destructive Interference

For destructive interference the path difference between the interfering waves should be an odd multiple of $\lambda/2$.

$2\mu t \cos r - \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

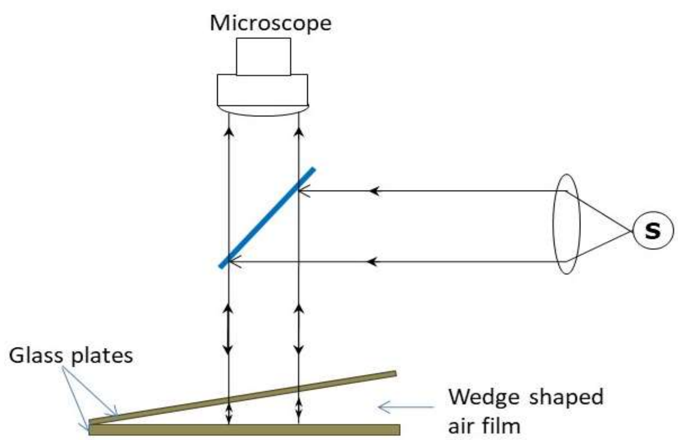
$2\mu t \cos r = 2n\frac{\lambda}{2} + \frac{\lambda}{2} + \frac{\lambda}{2}$

$2\mu t \cos r = n\lambda + \lambda$

$\mathbf{2\mu t \cos r = (n+1)\lambda}$ where n=0,1,2,3,…..

In general the film will appear dark by reflection when $2\mu t\cos r$ is an integral multiple of λ and the film will appear bright by reflection when $2\mu t\cos r$ is an odd multiple of $\lambda/2$.

Air Wedge

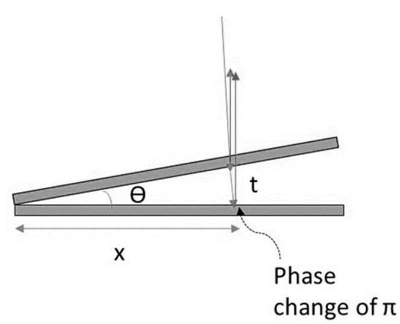


An air wedge is a wedge-shaped air film formed between two thin glass plates kept in contact at one edge.

When light falls normally on the first surface of the transparent thin air film, the incident wave is partially reflected and partially transmitted. The transmitted portion is then reflected from a second surface of air film.Thus, emerging from the thin film are two waves

(1) wave reflected from top surface of air film.

(2) wave reflected from lower surface of air film.



Wave reflected from lower surface of thin film have an additional path difference of $\frac{\lambda}{2}$ due to the refection at air to glass boundary. Therefore, path difference between two rays getting reflected from upper and lower surfaces of air wedge is given by

$\Delta = 2\mu t \cos r + \frac{\lambda}{2}$

For small values of r, cosr=1

$\Delta = 2\mu t + \frac{\lambda}{2}$

From figure $t = x \tan\theta$

$\Delta = 2\mu x \tan\theta + \frac{\lambda}{2}$

Condition to observe bright fringe is that path difference should be an integral multiple of λ.

$2\mu x \tan\theta + \frac{\lambda}{2} = n\lambda$

$2\mu x \tan\theta = n\lambda - \frac{\lambda}{2}$

$2\mu x \tan\theta = (2n-1)\frac{\lambda}{2}$

Condition to observe dark fringe is that path difference should be an odd multiple of λ/2

$2\mu x \tan\theta + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$

$2\mu x \tan\theta = 2n\frac{\lambda}{2} + \frac{\lambda}{2} - \frac{\lambda}{2}$

$2\mu x \tan\theta = n\lambda$; $n = 0,1,2,3..$ .gives the value of x where dark fringes are formed.

Let n^{th} dark fringe is formed at x_1 and $(n+m)^{th}$ dark fringe is formed at x_2. Then

$x_1 = \frac{n\lambda}{2\mu \tan\theta}$ $\quad$ $x_2 = \frac{(n+m)\lambda}{2\mu \tan\theta}$

$x_2 - x_1 = \frac{(n+m)\lambda}{2\mu \tan\theta} - \frac{n\lambda}{2\mu \tan\theta}$

$x_2 - x_1 = \frac{m\lambda}{2\mu \tan\theta}$

Band width/Fringe width is given by

$\beta = \frac{x2 - x1}{m}$

$\beta = \frac{\lambda}{2\mu \tan\theta}$

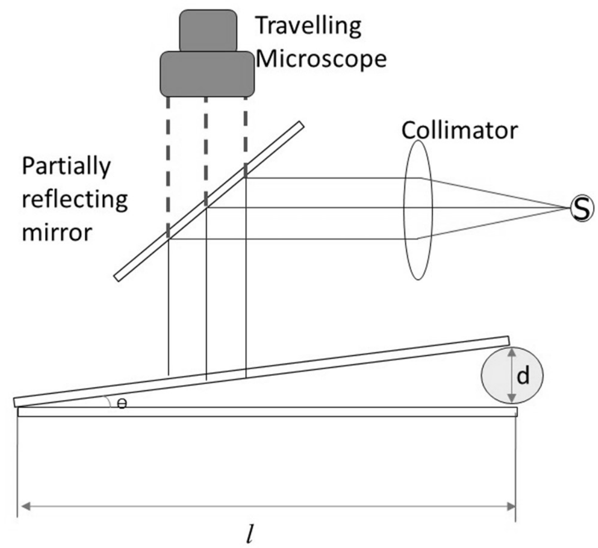
For air μ=1

$\beta = \frac{\lambda}{2 \tan\theta}$

Note: The change from dark to bright in the interference pattern is due to the change in the thickness of the film. Since the locus of all points having the same thickness of the film is a straight line, we get straight line fringes. Since the variation of thickness is uniform, fringes of equal thickness are observed in air wedge.

Measuring the diameter of a thin wire

A thin wire is wound near one end of a glass pate. It is kept over another glass plate so that they are in contact along one edge and a wedge-shaped air film is formed between them. Experimental arrangement to observe the interference pattern is shown in the figure. A parallel beam of monochromatic light falls on the air wedge to form interference pattern that can be observed using the travelling microscope.



Let 'l' be the length of the glass plate and 'd' be the diameter of the wire. From figure

$$\tan\theta = \frac{d}{l}$$

For air wedge $\beta = \frac{\lambda}{2\tan\theta}$

$$\beta = \frac{\lambda}{2\frac{d}{l}}$$

$$\beta = \frac{l\lambda}{2d}$$

Diameter of the wire is given by $d = \frac{l\lambda}{2\beta}$

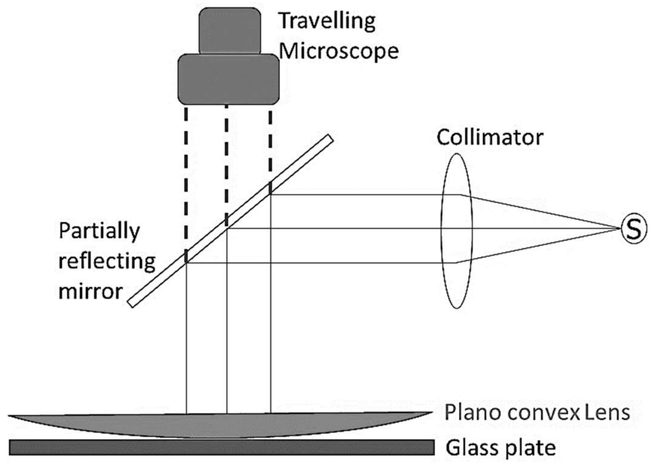
<u>Procedure</u>

1. Measure the length(l)of the glass plate by noting down positions the travelling microscope at both edges of the glass plate.
2. Measure the lengths x_1 and x_2 by noting the positions n^{th} and $(n+m)^{th}$ dark fringes.
3. Calculate the bandwidth $\beta = \frac{x2-x1}{m}$
4. Calculate the diameter of the wire using equation $d = \frac{l\lambda}{2\beta}$

Newton's Rings

Newton's rings apparatus consists of an optically plane glass plate on which a long focus convex lens is placed. A thin film is formed between the glass plate and the convex surface of the lens. Light from a monochromatic source (like sodium vapour lamp) is collimated using a lens and is allowed to fall on a partially reflecting mirror which is kept at 45° angle. Light gets reflected and falls normally on the lens-glass plate arrangement.

Light is reflected from the upper surface and lower surface of the thin film formed between glass plate and convex surface of lens. These rays interfere and a number of concentric dark and bright rings are formed, which are called Newton's rings. These rings can be observed though a microscope arranged vertically above the glass-plate.



Theory

In the figure LOL' represents the planoconvex lens with a radius of curvature 'R'. Lens is in contact with glass plate at point O. Let 't' be the thickness of the film a distance 'r' from O.

Using intersecting chord theorem

$AC \times CB = DC \times CO$

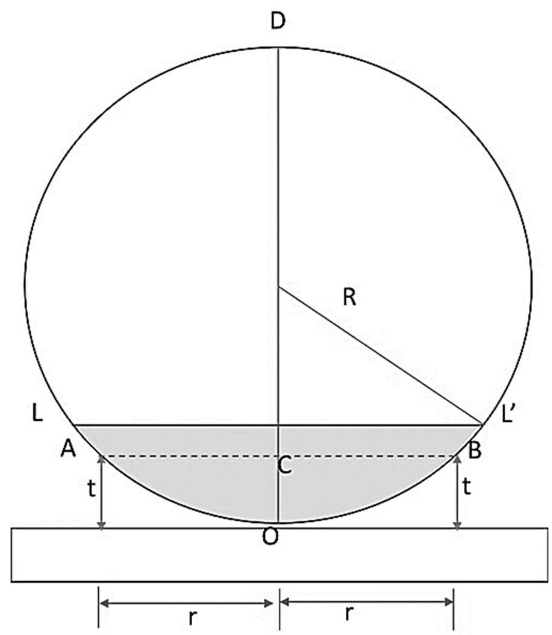
$r^2 = (2R - t)\ t$

$r^2 = 2Rt - t^2$

Neglecting t^2

$r^2 = 2Rt$

$2t = \frac{r^2}{R}$ ------------------------------(1)



The path difference between the rays reflected from top and bottom surfaces of the air film where thickness is 't' is given by

$\Delta = 2\mu t \cos r + \frac{\lambda}{2}$

For normal incidence; r = 0, cosr =1 and for air μ=1

$\Delta = 2t + \frac{\lambda}{2}$ -----------------------------(2)

Diameter of Bright Ring

Condition to observe bright ring is that path difference should be an integral multiple of λ

$2t + \frac{\lambda}{2} = n\lambda$

$2t = n\lambda - \frac{\lambda}{2}$

$$2t = (2n-1)\frac{\lambda}{2}$$

Substituting from eqn(1)

$$\frac{r_n^2}{R} = (2n-1)\frac{\lambda}{2}$$

$$r_n^2 = R(2n-1)\frac{\lambda}{2}$$

If d_n is the diameter of n^{th} bright ring

$$r_n^2 = \frac{d_n^2}{4}$$

$$\frac{d_n^2}{4} = R(2n-1)\frac{\lambda}{2}$$

$$d_n^2 = 2R(2n-1)\lambda$$

$$d_n = \sqrt{2R(2n-1)\lambda} \quad \text{where n=1,2,3,…}$$

Diameter of Dark Ring

Condition to observe dark ring is that path difference should be an odd multiple of λ/2.

$$2t + \frac{\lambda}{2} = (2n+1)\frac{\lambda}{2}$$

$$2t = 2n\frac{\lambda}{2} + \frac{\lambda}{2} - \frac{\lambda}{2}$$

$$2t = n\lambda$$

Substituting from eqn(1)

$$\frac{r_n^2}{R} = n\lambda$$

$$\mathbf{r_n^2 = Rn\lambda}$$

If d_n is the diameter of n^{th} dark ring

$$\frac{d_n^2}{4} = Rn\lambda$$

$$d_n^2 = 4Rn\lambda$$

$$d_n = \sqrt{4Rn\lambda} \quad \text{where n=1,2,3…}$$

Note: If the thin film was made of liquid of refractive index μ, then $d_n^2 = \frac{4Rn\lambda}{\mu}$

Since the locus of all points having the same thickness of the film is circle, we get circular interference pattern. In and around the center where glass plate and lens are in contact, the thickness air film is negligible, but the path difference resulting from the reflection at lower surface will be present. As a result the center spot will be always dark, irrespective of wavelength of light used.

Determination of Wavelength of Light

Let d_n and d_{n+k} are diameters of n^{th} and $(n+k)^{th}$ dark rings respectively

$$d_n^2 = 4Rn\lambda$$

$$d_{n+k}^2 = 4R(n+k)\lambda$$

$$d_{n+k}^2 - d_n^2 = 4R(n+k)\lambda - 4Rn\lambda$$

$$d_{n+k}^2 - d_n^2 = 4Rk\lambda$$

$$\lambda = \frac{d_{n+k}^2 - d_n^2}{4kR}$$

Diameter of n^{th} and $(n+k)^{th}$ rings can be measured using a travelling microscope and knowing the value of R, wavelength of light can be calculated

Determination of Refractive Index of a Liquid

Let d_n and d_{n+k} are diameters of n^{th} and $(n+k)^{th}$ dark rings respectively with air film

$$d_{n+k}^2 - d_n^2 = 4Rk\lambda$$

Let d'_n and d'_{n+k} are diameters of n^{th} and $(n+k)^{th}$ dark rings respectively with liquid in between plano-convex lens and plane glass plate. If μ is the refractive index of the liquid,

$$d'^2_{n+k} - d'^2_n = \frac{4Rk\lambda}{\mu}$$

$$\frac{d_{n+k}^2 - d_n^2}{d'^2_{n+k} - d'^2_n} = \frac{4Rk\lambda}{\frac{4Rk\lambda}{\mu}}$$

$$\mu = \frac{d_{n+k}^2 - d_n^2}{d'^2_{n+k} - d'^2_n}$$

Diameter of n^{th} and $(n+k)^{th}$ rings with air and liquid film can be measured using a travelling microscope and hence the refractive index of liquid can be determined.